Note that in either the lossy or lossless case a single arbitrary point inside the image circle is all that is necessary to completely calibrate the junction provided the corresponding Z_L is known.[5] However, a pure reactive termination is often more convenient and only two values are necessary (other than the point $x_L = \infty$ or $\Gamma_L = \Gamma_L' = +1$) as pointed out in Section III of our paper. Neither of these examples utilizes the technique of the cotangent method discussed in Section IV which is one of the most prominent features of the paper, particularly when dealing with lossy structures. It provides increased accuracy brought about by plotting a mean straight line from data corresponding to several reactances at the output port. Also note that in the lossless case of example 5, the calibration and solution are completely analytic when the original data are taken as absolute and the reference is chosen properly so that $Z_{in} = Z_L'$.

Stock and Kaplan also maintain that our method is less useful. Since they do not elaborate further we are unable to remark without specific examples. We do want to clearly state that although we do not claim our method to be a cure-all, the best, or any terms of such superlatives, we do believe it is a convenient, useful and accurate method of calibrating a junction.

Lastly, we acknowledge the typographical error and omission in our paper and agree that footnote 15 should read:

"The necessary and sufficient conditions for positive reality is that $R_{11} > 0$, or $R_{22} > 0$ and $R_{11}R_{22} - R_{12}^2 > 0 \cdots$." We also agree that the "Hyperbolic Protractor" booklet should have been mentioned in our references.

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[5] The iconocenter is a special case and corresponds to $Z_L = Z_{02}$. Deschamps has gone to great length for devising methods of constructing this point.

Messrs. Stock and Kaplan's Reply[6]

We are most deeply indebted to Messrs. Mittra and King for their worked out examples comparing Deschamps' method with theirs, as well as for pointing out misprints in the hyperbolic protractor pamphlet. The object of our note was not to imply that the Mittra-King method should not have been published, but to point out that it is one among many equally valid and simple techniques for two-port calibration. Specifically, it is an alternate method to that of Deschamps for some problems. The choice of a given method is subjective, and while we realize the linearization technique applied to the determination of two-port input impedance and to the Weissfloch method is a contribution, we feel that the method of Deschamps is simpler in allowing one to work graphically with plotted data as would be obtained from a loss circle measurement. In fact, for determination of input impedance for a given load impedance of a two-port, the methods of de Buhr[7] or Bolinder,[8] seem simplest, involving a calculation of only an interative impedance plus a graphical construction. Indeed, the simplest calculation of two-port input-output relations involves only the use of the cross-ratio. For lossless networks, Bracewell's[9] nomograph is the simplest method of determining the input(output) impedance corresponding to a known output(input) impedance though the further extension by Hinckelmann[10] is not that simple.

In this connection, it may be pointed out that Deschamps' hyperbolic distance is only the logarithm of a cross-ratio, so may be calculated as accurately as one pleases from data. It should be further pointed out that the three-point method is not inherent in the geometrical technique; in fact, the virtue of the technique is that constructions may be made directly on a unit circle containing plotted data. It is not necessary to expand the loss circle once plotted.

We further note that in problem 5 as done by Mittra-King they do not correct their answer for the 71° phase shift. While reference planes are arbitrary for an illustrative problem, an actual network may have definite reference planes.

The iconocenter, contrary to footnote 4 of the Mittra-King rebuttal is simple to construct: one applies the butterfly construction to the intersection of chords connecting quarter-wave separated data points. Its great value is that it represents the transform of a perfect load, obtained without the necessity of having such a load.

There are a number of problems in which all the loss circle methods become increasingly cumbersome, *e.g.*, in the three- and four-ports much more work is needed here to devise convenient measurement methods. Stein[11] has illustrated well the magnitude of these problems.

[6] Received June 12, 1962.

[7] J. de Buhr, "Eine neue Methode zur Bearbeitung linearer Vierpole," *FTZ*, vol. 8, pt. I, pp. 200–204, April, 1955; pt. II, pp. 335–340, June, 1955.
——, "Die zeichnerische Bestimmung der geometrischen Kenngrossen verlustloser, linearer Vierpole," *AEU*, vol. 9, pp. 350–354; August, 1955.
——, "Die geometrische Darstellungsweise kombinierter linearer Vierpole," *AEU*, vol. 9, pp. 561–570, December, 1955.
——, "Die geometrische Darstellungsweise des Parallel- und des Serienblind-widerstandes als verlustfreie sogenannte parabolische Vierpole," *Nachrtech. Z.*, vol. 8, pp. 636–641; December, 1955.
——, "Die geometrische Vierpol-Darstellung des Doppeltransformators," *AEU*, vol. 10, pp. 45–49; January, 1956.
——, "Die geometrische elementarste Darstellungsform verlustloser linearer Vierpole," *Nachrtech. Z.*, vol. 9, pp. 80–84; February, 1956.
——, "Die geometrische Darstellungsweise hintereinander geschalteter allgemeiner, verlustbehafteter Vierpole," *AEU*, vol. 11, pp. 173–176, April, 1957.

[8] E. F. Bolinder, "Impedance and Power Transformation by the Isometric Circle Method and Non-Euclidean Hyperbolic Geometry," Radiation Lab., M.I.T., Cambridge, Rept. No. 312; June 14, 1957.

[9] R. N. Bracewell, "A new transducer diagram," PROC. IRE, vol. 42, pp. 1519–1521; October, 1954.

[10] O. Hinckelmann, "Graphical method for transforming impedances," IRE TRANS. ON MICROWAVE TECHNIQUES, vol. MTT-10, pp. 139–141; March, 1962.

[11] S. Stein, "Graphical analysis of measurements on multi-port waveguide junctions," PROC. IRE (*Correspondence*), vol. 42, p. 599; March, 1954.

Messrs. Mittra and King[12]

There never has been any question in the mind of the present authors that Prof. Deschamps' geometric viewpoints are outstanding contributions to the theory of Microwave Measurements. They also agree with Kaplan and Stock that there are several methods which possibly provide simple graphical means for relating input and output impedance through a junction; although the word "simple" must be used in a subjective sense in relation to the different methods. We should like to point out again that the main emphasis in our paper is the averaging technique for smoothing out the errors.

Regarding their criticism of our workout of Problem 5, it should be pointed out that there was the implication in deriving the solution that the problem was stated after the reference plane shift, and hence no correction was necessary in the solution.

As regards the iconocenter, it is our experience that its determination for a particular set of experimental data is difficult when the experimental intersection points are distributed over a region due to the presence of experimental errors in the measurements. This is particularly true when the discontinuity being measured is small. We agree with Kaplan and Stock that the actual construction of the iconocenter procedure is straightforward if the exact values of Γ_{in} are known, but, of course, in practice such is not the case as finite errors are obviously present. Indeed, these were the very reasons why the Linear Transformation Method was developed.

We hope that we have made our position on the major issues sufficiently clear through the two communications and that this will end further discussions along similar lines.

[12] Received July 12, 1962.

On "A Solid-State Microwave Source from Reactance-Diode Harmonic Generators"*

In a recent paper,[1] Hyltin and Kotzebue have given some interesting formulas about the maximum efficiency of reactance-diode harmonic generators. It may be interesting to submit to the attention of the authors some considerations about their theoretical analysis.

1) Formula (11) and the consequent results were obtained by Hyltin and Kotzebue through a matrix inversion, starting from the hypothesis of a voltage controlled variable capacitor. It is also possible to arrive at the same result, more directly, starting from the dual case of a charge controlled variable elastance $S_d(q)$ (Fig. 1), which may be ex-

* Received March 1, 1962.

[1] T. M. Hyltin and K. L. Kotzebue, IRE TRANS. ON MICROWAVE THEORIES AND TECHNIQUES, vol. MTT-9, pp. 73–78, January, 1961.

pressed as a function of the charge q_1 only, present across the variable capacitor at the fundamental angular frequency ω_1:

$$s(q_1) = S_0 \sum_{p=-\infty}^{+\infty} \beta_p{}^{jp\omega_1 t}, \qquad (\beta_p = \beta_{-p}). \quad (1)$$

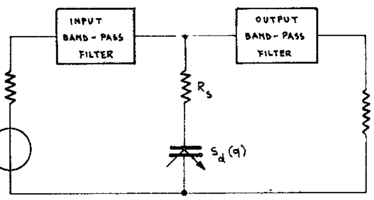

Fig. 1—Equivalent circuit of an harmoic generator with a charge controlled varactor diode.

Starting from the same small signal approximation and following a procedure, similar to the one followed in Hyltin and Kotzebue's analysis, we may write the voltage across the variable capacitor as a function of $s(q_1)$ and of the charge $q = q_1 + q_2$:

$$\frac{dv}{dt} = s(q)\frac{dq}{dt} \cong s(q_1)\left(\frac{dq_1}{dt} + \frac{dq_2}{dt}\right) + \left.\frac{ds}{dq_1}\right|_{q_1} q_2\left(\frac{dq_1}{dt} + \frac{dq_2}{dt}\right)$$

$$\cong s(q_1)\left(\frac{dq_1}{dt} + \frac{dq_2}{dt}\right) + \left.\frac{ds}{dt}\right|_{q_1} q_2. \quad (2)$$

Using exponential representation we may write

$$q_1 = Q_1{}^{i\omega_1 t} + Q_1{}^* e^{-j\omega_1 t}, \qquad Q_1 = Q_1{}^*$$

$$q_2 = Q_n e^{jn\omega_1 t} + Q_n{}^* e^{-jn\omega_1 t}, \quad \phi_n = \arg Q_n,$$

$$i_1 = \frac{dq_1}{dt} = j\omega_1 Q_1 e^{j\omega_1 t} - j\omega_1 Q_1{}^* e^{-j\omega_1 t},$$

$$i_2 = \frac{dq_2}{dt} = jn\omega_1 Q_n e^{jn\omega_1 t} - jn\omega_1 Q_n{}^* e^{-jn\omega_1 t}. \quad (3)$$

By considering in (2) the terms which are functions only of $e^{j\omega_1 t}$ and $e^{jn\omega_1 t}$, we may write the following equations, relating V_1 and V_n to I_1 and I_n:

$$V_1 = \frac{S_0}{j\omega_1}(1 - \beta_2)I_1 + \frac{S_0}{jn\omega_1}(\beta_{n-1} + \beta_{n+1}e^{-j2\phi_n})I_n,$$

$$V_n = \frac{S_0}{jn\omega_1}(\beta_{n-1} - \beta_{n+1})I_1 + \frac{S_0}{jn\omega_1} I_n. \quad (4)$$

Voltage V_n consists of the sum of two terms, the first one being proportional to Q_1 and the second one being proportional to Q_n and having the same phase.

It is possible to show[2] that, in order to obtain maximum efficiency, the second term must have a phase shift of $+\pi/2$ with respect to the first; this means that the output circuit must be tuned in order to have $\phi_n = \pi/2$. Then it is possible to show[1-3] that maximum efficiency loading conditions are

$$R_g = R_L = R_S\sqrt{1 + x^2}, \quad (5)$$

where x may be expressed as a function of the diode's Q and of a distortion factor $\bar{\beta}$ (following Hyltin and Kotzebue):

$$x = \bar{\beta} Q_n,$$

$$\bar{\beta} = \beta_{n-1} - \beta_{n+1}, \qquad Q_n = \frac{S_0}{n\omega_1 R_S}; \quad (6)$$

of, if we prefer, as the ratio between a characteristic diode's angular frequency, ω_{cn}, as n-harmonic generator, and the fundamental angular frequency ω_1:[2]

$$x = \frac{\omega_{cn}}{\omega_1}$$

$$\omega_{cn} = \frac{S_0}{R_S}\,\frac{\beta_{n-1} - \beta_{n+1}}{n}. \quad (6a)$$

Maximum transducer gain g_T (*i.e.*, maximum conversion efficiency) may be written as

$$g_T = \frac{\sqrt{1 + x^2} - 1}{\sqrt{1 + x^2} + 1} = \frac{x^2}{(\sqrt{1 + x^2} + 1)^2}. \quad (7)$$

2) Expression (7) is similar to expression (17) in Hyltin's and Kotzebue's paper, but as we allow the loss to approach zero (by making Q, or x, very large) the transducer gain approaches unity. On the contrary said expression (17) may never quite reach unity.

This may not be attributed to the initial small signal approximation, as the authors believe. In fact the two hypotheses (high Q and small-signal) may coexist independently: harmonic generators may operate also with small signals and their conversion losses must be zero, provided the diode is lossless. The discrepancy is due to a small mistake, escaped to the authors, who forgot, in the first of their equations (9), an additional term, proportional to $\gamma_{n+1}V_n{}^*$. Taking into account this term, the correct expression is found for the transducer gain, which coincides with the above expression (7); similarly the correct expression for the parameter $\bar{\gamma}$ is

$$\bar{\gamma} = \frac{\gamma_{n-1} - \gamma_{n+1}}{1 - \gamma_2},$$

which differs from $\bar{\beta}$ expression (6) in having $1 - \gamma_2$ at the denominator.

3) Expressions (6) and (7) are derived starting from the hypothesis $q_2 \ll q_1$ and $i_2 \ll i_1$. If the efficiency is high ($g_T \cong 1$), because of the condition (5), i_2 and i_1 must be practically equal. Therefore Hyltin and Kotzebue's analysis doesn't seem useful in the most interesting cases of high efficiencies.

Nevertheless it may be interesting to observe that, if we use expressions (6) and (7) to calculate the transducer gain for an ideal diode, having a series resistance R_S and whose elastance has an abrupt variation, between a maximum value S_{max} and zero, (Fig. 2), we can find that ω_{cn} (if for simplicity we assume only even values of n) may be written as

$$\omega_{cn} = \frac{2}{\pi}\,\frac{\omega_c}{n^2 - 1}, \quad (8)$$

where

$$\omega_c = \frac{S_{max}}{R_S}. \quad (9)$$

Expression (8) is the same found elsewhere[2] rigorously, taking into account also the large signal case.

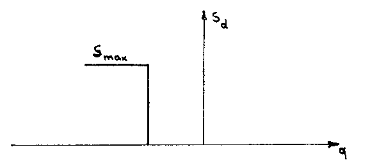

Fig. 2—Differential elastance vs applied charge for an ideal diode.

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Authors' Comment[4]

The authors are indebted to Mr. Stracca for correctly pointing out an error in the harmonic generator analysis of Hyltin and Kotzebue. Fortunately the correction is small and changes the published curves by less than 1 db.

Two points raised by Stracca should perhaps be briefly commented upon. The first is a possible misunderstanding of the phrase "small signal" as used by Hyltin and Kotzebue. This referred to the assumption that the harmonic components were small compared to the input signal, and therefore this might be termed a low-efficiency analysis rather than a small-signal analysis.

A second point raised by Stracca is in regard to the usefulness of the authors' analysis in light of the restriction to low efficiency. This point occurred to the authors when the analysis was originally formulated; it has in fact been somewhat surprising to note the close agreement between the analysis and experimental results. For example, the curves presented[1] have been in use at Texas Instruments, Inc., for approximately three years and have successfully predicted experimental results over a range of conversion efficiencies from -2 db to -20 db. In this connection it should be noted that some degree of empirical selection of the drive parameter a is needed.

Such an approach also appears to yield good results for situations not strictly covered by the model used in the analysis. For example, it has been found that some diffused-junction silicon diodes have capaci-

[2] G. B. Stracca, "Generazione di armoniche con diodi varactor," *Alta Frequenza*, vol. 31, pp. 134–149, March, 1962; also, (in English) *Alta Frequenza*, vol. 31, pp. 294–307, May, 1962.

[3] K. M. Johnson, "Large signal analysis of a parametric harmonic generator," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-8, pp. 525–532; September, 1960.

[4] Received June 4, 1962.

tance changes in excess of that predicted from the inverse-cube-root variation of the deplc-tion-layer capacitance. This is attributed to stored charge carriers under conditions of forward bias. Thus the harmonic generation efficiency of these diodes is above that predicted for cube-root capacitance variation. However, it is interesting to note that agreement between experiment and theory can be obtained for at least one published result using these diodes,[5] using Fig 4 of Hyltin and Kotzebue which is for square-root capacitance variation. The result is that of Lowell and Kiss who reported on a fifth-harmonic and eighth-harmonic generator. Using their data on the diodes used, and assuming that the effective diode capacitance at operating bias is 0.6 the zero bias values, we obtain predicted efficiencies of about 5 db for the fifth harmonic circuit and about 19 db for the eighth harmonic circuit. The values reported by Lowell and Kiss are aout 5.5 db and 19 db, respectively.

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[5] R. Lowell and M. J. Kiss, "Solid-state microwave power sources using harmonic generation," PROC. IRE, vol. 48, pp. 1334-1335; July, 1960.

Double-Layer Matching Structures*

In the millimeter wavelength region, as in other regions, the design of matching layers for dielectric surfaces is limited by the lack of suitable dielectric materials. Artificial dielectric layers, formed by periodic perturbations of the boundary surface, are not practical because of the small physical dimensions required. The following approach uses two layers of materials whose relative dielectric constants are given. The thicknesses of the layers are chosen to eliminate reflections at the desired center frequency. A broad-band match is obtained because the layers can be made less than an eighth wavelength in thickness.

In the case of normal incidence upon lossless dielectric layers, transmission line theory may be used to determine the matching conditions. Refering to Fig. 1, and assuming that the first section is matched,

$$Z_4 = Z_3 \frac{Z_2(Z_1 + jZ_2 \tan\theta_2) + jZ_3 \tan\theta_3(Z_2 + jZ_1 \tan\theta_2)}{Z_3(Z_2 + jZ_1 \tan\theta_2) + jZ_2 \tan\theta_3(Z_1 + jZ_2 \tan\theta_2)} \quad (1)$$

at the center frequency. The characteristic impedance and electrical length of the jth section are Z_j and θ_j, respectively.

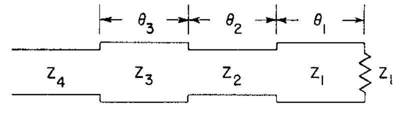

Fig. 1—Cascaded transmission line system.

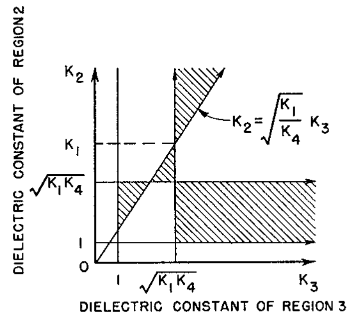

Fig. 2—Allowed values of relative dielectric constants of regions 2 and 3.

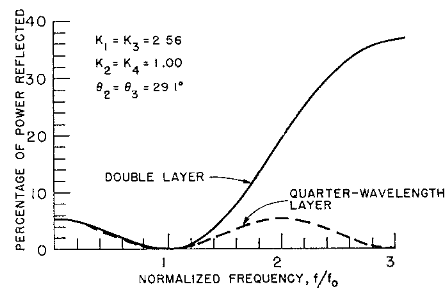

Fig. 3—Reflected power as function of normalized frequency.

Equating real and imaginary parts of (1) gives two equations which may be solved for θ_2 and θ_3

$$\theta_2 = \tan^{-1}\left[\frac{n_2^2(n_1n_4 - n_3^2)(n_4 - n_1)}{(n_2^2 - n_1n_4)(n_2^2n_4 - n_1n_3^2)}\right]^{1/2} \quad (2)$$

$$\theta_3 = \tan^{-1}\left[\frac{n_3^2(n_2^2 - n_1n_4)(n_4 - n_1)}{(n_1n_4 - n_3^2)(n_2^2n_4 - n_1n_3^2)}\right]^{1/2}. \quad (3)$$

The refractive index of the jth layer is $n_j = \sqrt{K_j}$, where K_j is the relative dielectric constant of the layer.

In a typical problem, K_1 and K_4 are specified. It may be shown that for $K_1 > K_4$, the values of K_2 and K_3 that yield real values of θ_2 and θ_3 lie in the shaded regions shown in Fig. 2. A practical design can usually be found using only available low-loss dielectric materials.

As an example, consider the problem of matching a polystyrene-air interface. Letting $K_1 = 2.56$ for polystyrene and $K_4 = 1$ for air, it is seen that one solution is

$$K_2 = 1.00$$

$$K_3 = 2.56$$

$$\theta_2 = \theta_3 = 29.1°.$$

Fig. 3 shows ratio of reflected power to incident power for this solution and for a quarter-wavelength layer designed for the same center frequency. The bandwidths of the two structures are comparable.

Since thin films of most plastics are commercially available, the double layer matching structure is feasible at millimeter wavelengths. One such structure using polystyrene film and polystyrene foam sheet on polystyrene has proved successful at 70 Gc.

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Microwave Noise-Figure Measurement for Small Noise Output*

The purpose of this communication is to propose a new method of noise-figure measurement for a microwave amplifier of small noise output. The new method is helpful when the accuracy of a conventional method is not satisfactory.

In conventional measurements, when the noise output of the microwave amplifier is too small for direct noise power measurement, an auxiliary receiver is used after the amplifier under test. The noise figure of the amplifier, F_1, is given by[1]

$$F_1 = F_{12} - \frac{F_2 - 1}{G_1}. \quad (1)$$

In this equation,

F_{12} = noise figure of over-all system,
F_2 = noise figure of the auxiliary receiver,
G_1 = gain of the amplifier under test.

It is possible to express F_1, F_2, G_1, and F_{12} in the following manner:

$$\left.\begin{array}{l} F_1 \equiv f_1 \times 10^{n_1},\ F_2 \equiv f_2 \times 10^{n_2}, \\ G_1 \equiv g \times 10^{m} \end{array}\right\} \quad (2)$$

and

$$F_{12} \equiv f_{12} \times 10^{n_{12}}$$

where $0 < (f_1, f_2, f_{12} \text{ or } g) > 10$ and n_1, n_2, m and n_{12} are positive integers. Then, when $F_2 \gg 1$, (1) can be rewritten as

$$F_1 \doteq f_{12} \times 10^{n_{12}} - \frac{f_2}{g} \times 10^{n_2 - m}$$

$$= \left(f_{12} - \frac{f_2}{g} \times 10^{n_2 - m - n_{12}}\right) \times 10^{n_{12}} \quad (3)$$

* Received April 2, 1962; revised manuscript received, May 18, 1962. This is a part of research which is jointly supported by Frederick Gardner Cottlell Grant and Marquette University Committee on Research Grants.

[1] H. T. Friis, "Noise figures of radio receivers," PROC. IRE, vol 32, pp. 419-422; July, 1944.